Calculation of Single Conductor Capacitance by Solving the Electrostatic Field

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Abstract — The capacitance between two terminals of a single conductor working with time-dependent signals is defined by the amount of energy stored in the electric field outside the conductor. A simple approach for calculating the capacitance is presented in this paper, which only needs the computation of an electrostatic field. The approach is derived based on two assumptions, (1) the distribution of potentials on the conductor surface is almost the same, created by a time-dependent current and a direct current flowing in the conductor, (2) the distribution of the potential created by a direct current in the conductor can be modeled by an electrostatic field, in which the conductor is replaced by a dielectric with high permittivity. The approach is only suitable for low-frequency situations, where the displacement current and the inductive electric field can be disregarded.

I. INTRODUCTION

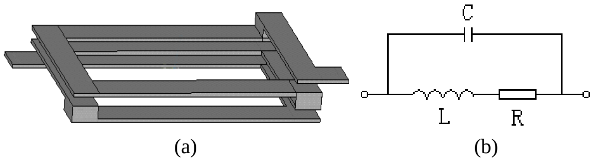
The time-dependent equivalent circuit of a single conductor, such as the coil shown in Fig. 1 (a), is a capacitor connected in parallel with a R-L branch, as shown in Fig. 1 (b). The calculation of the capacitance is essential for its equivalent circuit modeling. The value of the three circuit elements can be obtained simultaneously by calculating a time-dependent electromagnetic field, especially the electric field in the air region, which is not provided by $\boldsymbol{A}$-φ method. Therefore, based on the current algorithms, it is not easy to get the solution of the capacitance. A simplified approach for calculating the capacitance low frequency cases is presented in this paper, which only needs the computation of an electrostatic field.

II. DEFINITION OF CAPACITANCE FOR SINGLE CONDUCTOR

The capacitance between two terminals of a conductor working with time-dependent signals is related to the electric energy W_e outside the conductor, whose value is defined as

$$C = 2W_{\mathrm{e}} / U^2 , \qquad (1)$$

where U is the voltage between the two terminals. Therefore, to calculate the capacitance, the electric energy outside the conductor should be obtained under the incitation of U.



(a) (b)

Fig. 1. A coil and its circuit model working with time-dependent signals.

III. A SIMPLIFIED APPROACH FOR THE CALCULATION OF THE CAPACITANCE

The value of the capacitance is almost the same for signals with different frequency, even to direct current (DC) signal, since the voltage drop of the conductor surface along the current flow is almost the same in different situations. Therefore, the calculation of the energy or the capacitance can be conducted in DC situations, although a capacitor in the equivalent circuit model of a single conductor does not make sense for a DC signal. For calculating the capacitance, the potential on the conductor surface should be determined first, which generally requires a solution of the eddy current problem. Then the electrostatic field outside the conductor is calculated by employing the potential as the boundary condition..

Actually, it is not necessary to conduct the calculation in such a two-step procedure. We propose a one-step approach that only needs the solution of an electrostatic field. This approach combines the regions outside and inside the conductor into account together to form an electrostatic field domain by replacing the conductor with a dielectric with high permittivity. A voltage is set on the two terminals, or two end-surfaces, of the conductor. After the solution, only the energy outside the conductor region is taken to calculate the capacitance. The calculation is easily implemented using the finite element method.

The approach makes use of the advantage of a formal analogy between the electrostatic and the steady current fields. That is the distribution of the electric displacement in the conductor region, obtained by calculating the electrostatic field, is almost the same as the distribution of the current density obtained by calculating the current field. It is necessary to set a large value of permittivity in the conductor region, relative to the permittivity of the material surrounding the conductor, to fulfill the formal analogy between the fields.

Fig. 2 gives the distribution of the potentials along the external edges from one terminal to another, created by the DC field and the electrostatic field with relative permittivity of 100 and 1.e6. The maximum difference of the potentials at the same points, created by the DC field and the electromagnetic field with relative permittivity of 1.e6 is only 1.19 mV.

The degree of the formal analogy between the two fields can be also verified indirectly by checking the identical degree of the electric displacement flux on different cross sections, since the current density flux, i.e., the total current on every section must be the same. Three sections were chosen to check the electric displacement flux, which are one end surface, a section on the middle of the long side and another on the middle of the short side. Table I gives the value of the displacement flux on each section in two cases with relative permittivity of 100 and 1.e6.

From the results shown in Fig. 2 and Table I, we can see that the formal analogy of the fields can be fulfilled by setting a relative permittivity of 1.e6, and a relative permittivity of 100 is definitely too small. In our experience, a relative permittivity of not less than 1.e6 should be adopted, which can be seen from the example in Section V.

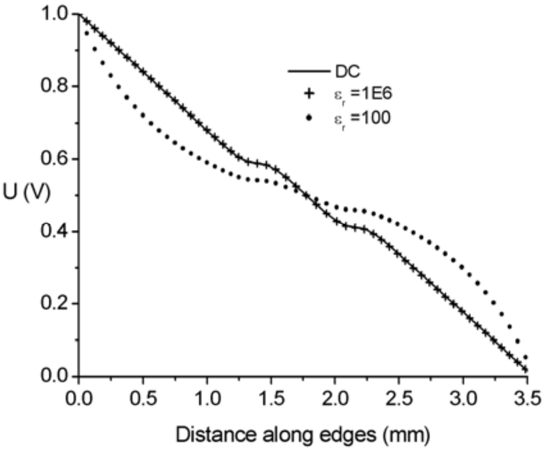


Fig. 2. Distribution of the potentials along the external edges in three situations.

TABLE I
ELECTRIC DISPLACEMENT FLUX ON THE THREE SECTIONS IN TWO CASES

Section number	1	2	3
Flux for ε_r =100	3.0322e-6	1.0177e-6	0.46792e-6
Flux for ε_r =1.e6	1.1325e-2	1.1322e-2	1.1329e-2

IV. CALCULATION EXAMPLE

The three parameters of a coil were calculated, whose structure is shown in Fig. 1 (a). The coil was made of silver. This type of coil is used as a resonance element in mobile telephones. Therefore, it is very important to calculate the capacitance and the inductance to obtain the resonance frequency in the design of the coil.

A. *Calculation Results*

Based on the one-step approach described above, the capacitance was calculated. The results with different relative permittivity set in the conductor region are given in Table II, from which we can see that the capacitance can be taken as 0.967 pF. The distribution of the electric displacement in the coil with the relative permittivity of 1.e6 is illustrated in Fig. 3.

Actually, the value of the inductance is also almost the same in the DC situation and time-dependent situations. It is easy to understand that the non-uniform distribution of the current on the cross section of the conductor influences the internal inductance, but has hardly any influence on the external inductance, while the internal inductance is much smaller than the external one. For the coil, the result of calculating total inductance in the DC situation is 19.671 nH, while for a frequency of 1 MHz it is 19.456 nH, which was calculated with the software that employs $\mathbf{A}-\varphi$ algorithm. Therefore, the inductance can be calculated by computing the magnetostatic field.

TABLE II
CALCULATION RESULTS OF CAPACITANCE AGAINST DIFFERENT PERMITTIVITY

ε_r	1.e7	1.e6	1.e5	1.e4	1.e3
Capacitance (pF)	0.9672	0.9671	0.8818	0.7975	0.5042

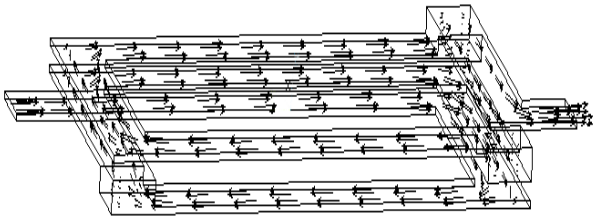


Fig. 3. Distribution of the electric displacement in the conductor region, obtained by the calculation of the electrostatic field.

Verification of the Results

According to calculation of the capacitance and the inductance obtained in the steady current situation, the resonance frequency of the coil is 1.155 GHz, while the actual measured value is 1.24 GHz. At the resonance frequency, the calculation result of the resistance is 5.960 Ohm, which was calculated with the software that employs $\mathbf{A}-\varphi$ algorithm. Based on the measurement of the quality factor and the calculation result of the capacitance, the resistance of the coil can be obtained, which is 5.8 Ohm. The coil with more turns was also simulated by the approaches. The calculation results were also in accordance with the measurements.

V. CONCLUSION

The capacitance of a single conductor working in low frequency signals, where the inductive electric field can be disregarded, may be calculated by only estimating an electrostatic field, which replaces the conductor region by a dielectric with high permittivity. A relative permittivity of not less than 1.e6 should be set in the conductor region to get satisfactory results.

If the relative permittivity is set to a value less than 1.e10 and the PCG solver is employed, the large jump in permittivity does not influence the solution of the FEM significantly, although it creates stiffness matrixes with large condition numbers.

The inductance can be calculated by computing the magnetostatic field. The resistance must be calculated by solving the eddy current problem.